QCD Lecture-2

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Outline

Lecture -2 naive parton model -> et M⁻ Scattering -> e P clastic scattering L Form Factors -> EP Inclastic scattering 1 structure functions L Bjørken Scaling Les partons Distributions functions Louis QCD Improved poston model Beyord the scope of this leduxe.

Electron muon scattering we wish to evaluate the cross section in the cross section in the cross section in the frame, the frame, the f which the muon is initially at rest, *p* = *(M,* 0). The particle momenta in this frame To reach the last line, we have used the following kinematic relations: *q2"", -2k. k'* "'" *-2EE'(1* - cosO) ⁼ *da (20:E,)2* { ² *(J q2.* ² *(J}* (*q2*) *dE'dn* ⁼ *q4* cos "2 - *2M2* SIll"2 *^p* + *2M .* 8.2 Electron-Proton Scattering. Proton Form Factors

M.C. Kumar where *A* = 1 + *(2EIM*)sin2 and the step function *O(x)* is 1 if *^x* > 0 and

(6.44)

(6.49)

Electron proton elastic scattering *J'"* ⁼ *^e U(p')[]U(p) ei(p'-p).X,* **Exercise 8.6 Show that is not an independent scalar variable by the scalar variable**

expressing it in terms of the variable *q2.*

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$$
(1 + \kappa)e/2M,
$$

$$
\left[F_1(q^2)\gamma^{\mu} + \frac{\kappa}{2M}F_2(q^2) i\sigma^{\mu\nu}q_{\nu}\right]
$$

the current at the other end of the propagator. The most general form of the

Fig. 8.6 The ep -> eX cross section as a function of the missing mass *W.* Data are from

$$
\frac{d\sigma}{d\Omega}\Big|_{\text{lab}} = \left(\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\right)\frac{E'}{E}\left\{\left(F_1^2 - \frac{\kappa^2q^2}{4M^2}F_2^2\right)\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\left(F_1 + \kappa F_2\right)^2\sin^2\frac{\theta}{2}\right\}
$$

see (6.50). This is known as the Rosenbluth formula. The two form factors,

represented by the blob in Fig. 8.2. These form factors can be determined

and their coefficients are functions of *q2 (q2* is the only independent scalar

therefore be chosen so that in this limit is limited with in this limit in this limit is limited with α

Electron proton elastic scattering experimentally by measuring as a function of *0* and *q2.* Note that if the *q2* ()2 . 2 *⁰* } *- 2M² F)* ⁺ *KF2* SIn 2" '

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$$
G_E \equiv F_1 + \frac{\kappa q^2}{4M^2} F_2
$$

$$
G_M \equiv F_1 + \kappa F_2,
$$

(8.15)

$$
\frac{d\sigma}{d\Omega}\bigg|_{\text{lab}} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\frac{E'}{E}\bigg(\frac{G_E^2 + \tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2} + 2\tau G_M^2\sin^2\frac{\theta}{2}\bigg)
$$

the recoil of the proton makes this impossible. However, it is possible to show

as the charge and magnetic moment distributions of the 'proton. Unit of the 'proton. Unit of the 'proton. Unit
In the 'proton. Unfortunately, we can consider the 'proton. Unfortunately, we can consider the 'proton. Unfort

and their coefficients are functions of *q2 (q2* is the only independent scalar

$$
\tau \equiv -q^2/4M^2.
$$

Electron proton elastic scattering

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G*^E* and G*^M* are referred to as the electric and magnetic form factors, respec-

$$
\langle r^2 \rangle = 6 \left(\frac{d G_E(q^2)}{dq^2} \right)_{q^2=0} = (0.81 \times 10^{-13} \text{ cm})^2
$$

$$
G_E(q^2) \approx \left(1 - \frac{q^2}{0.71}\right)^{-2}
$$
 mean square proton charge radius

tion. Using the result of Exercise 8.4, we conclude that the charge distribution of

and their coefficients are functions of *q2 (q2* is the only independent scalar

The same radius of about 0.8 fm is obtained for the magnetic moment distribution for the magnetic moment distribu-

tor form that can be constructed from *p, p', q* and the Dirac y-matrices sandwiched between *u* and *u.* There are only two independent terms, y'" and *ia"'vqv* The behavior for small $\mathsf{R}\mathsf{e}\mathsf{r}$ is the residual term in the residual term $\mathsf{R}\mathsf{e}\mathsf{r}$ is the residual term $\mathsf{R}\mathsf{e}\mathsf{r}$ is the residual term $\mathsf{R}\mathsf{e}\mathsf{r}$ is the residual term $\mathsf{R}\mathsf{e}\mathsf{r}$ *r Ref : Halzen and Martin*

M.C. Kumar $W.C.$ Measured out by the proton vertex). The proton vertex $W.S.$ are ruled out by the conservation $W.S.$ look at its structure by increasing the - *q2* of the photon to give better spatial resolution. This can be done simply by requiring a large energy loss of the simply by requiring a large energy loss of the simply by requiring a large energy loss of the simply by requiring α tion. Using the result of $\mathbf{M}.\mathbf{C}$, we conclude that the charge distribution of $\mathbf{M}.\mathbf{C}$,

Electron proton inelastic scattering from the measurements. Figure 8.6 shows the invariant mass distribution. One I I *I W (GeV/c²)* Fig. 8.6 The ep -> eX cross section as a function of the missing mass *W.* Data are from the Standard Linear Acceptulator. The elastic scattering $\overline{}$

i
III

which is already summed and averaged over spins. We write α

notices the peak when the proton does not break up *(W* :::::: *M)* and broader peaks

 $-\mu$ ^r $\left($ - $\right)$

the Stanford Linear Accelerator. The elastic peak at *W* ⁼ *M* has been reduced by a factor

$$
d\sigma \sim L_{\mu\nu}^e (L^p)^{\mu\nu} \qquad \qquad d\sigma \sim L_{\mu\nu}^e W^{\mu\nu}
$$

$$
W^{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{M^2} p^{\mu} p^{\nu} + \frac{W_4}{M^2} q^{\mu} q^{\nu} + \frac{W_5}{M^2} (p^{\mu} q^{\nu} + q^{\mu} p^{\nu})
$$

the cross section vanishes after insertion into (8.23) because the tensor *L;"* is

Electron proton inelastic scattering re
Electronic modern includio co Thus, only two of the four inelastic structure functions of (8.24) are indepen- \blacksquare e-the same calculation for e-p. (or episodic substitution of \blacksquare tion of *J¥,.v'* given by (8.27), for *L;:'vuon* (or *Ltv)'* Using the expression (6.25) for *(Le)"v* and noting (8.25), we find

dent; so we may write

$$
q^{\mu}L_{\mu\nu}^{e} = q^{\nu}L_{\mu\nu}^{e} = 0, \qquad W_{5} = -\frac{p \cdot q}{q^{2}}W_{2},
$$
\n
$$
W_{4} = \left(\frac{p \cdot q}{q^{2}}\right)^{2}W_{2} + \frac{M^{2}}{q^{2}}W_{1}
$$
\n
$$
q W^{\mu\nu} = q W^{\mu\nu} = 0.
$$

(8.26)

 4μ '' \mathbf{Y} Thus, only two of the four intelastic structure functions of $\mathcal{O}(8,24)$ are independent functions of $(8,24)$

The proof may be left until after (8.39); it follows from *Jll.jll.* = O. As a

$$
W^{\mu\nu} = W_1 \bigg(-g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{q^2} \bigg) + W_2 \frac{1}{M^2} \bigg(p^{\mu} - \frac{p \cdot q}{q^2} q^{\mu} \bigg) \bigg(p^{\nu} - \frac{p \cdot q}{q^2} q^{\nu} \bigg)
$$

$$
q^2
$$
 and $v = \frac{p \cdot q}{M}$. $x = \frac{-q^2}{2p \cdot q} = \frac{-q^2}{2Mv}$, $y = \frac{p \cdot q}{p \cdot k}$

$$
\frac{d\sigma}{dE'd\Omega}\bigg|_{\text{lab}}=\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\left\{W_2(\nu,q^2)\cos^2\frac{\theta}{2}+2W_1(\nu,q^2)\sin^2\frac{\theta}{2}\right\},\
$$

variables

x=--=-- 2p· ^q 2Mp' (8.30)

$$
\left(\cos^2\frac{\theta}{2}-\frac{q^2}{2m^2}\sin^2\frac{\theta}{2}\right)\delta\left(\nu+\frac{q^2}{2m}\right).
$$

Electron muon scattering

$$
\left(\frac{G_E^2 + \tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2} + 2\tau G_M^2\sin^2\frac{\theta}{2}\right)\delta\left(\nu + \frac{q^2}{2M}\right), \quad \text{Elec}
$$

 \mathcal{F} Electron proton elastic scattering

$$
W_2(\nu, q^2)\cos^2\frac{\theta}{2}+2W_1(\nu, q^2)\sin^2\frac{\theta}{2}.
$$

where -r ⁼ *-q2/4M2* and Mis the mass of the proton. Finally, for the case when Electron proton inelastic scattering

Electron proton inelastic scattering

$$
2W_1^{\text{point}} = \frac{Q^2}{2m^2} \delta\left(\nu - \frac{Q^2}{2m}\right),
$$

$$
W_2^{\text{point}} = \delta\left(\nu - \frac{Q^2}{2m}\right).
$$

exchange is dominant. If the dominant ϵ

$$
\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\frac{E'}{E}\Big(\qquad \Big)
$$

For point like particles *e;* where *eq* is the quark's fractional charge),

$$
\left(\cos^2\frac{\theta}{2}-\frac{q^2}{2m^2}\sin^2\frac{\theta}{2}\right)\delta\left(\nu+\frac{q^2}{2m}\right).
$$

$$
Q^2 \equiv -q^2.
$$

$$
W_2(\nu, q^2) \cos^2 \frac{\theta}{2} + 2W_1(\nu, q^2) \sin^2 \frac{\theta}{2}.
$$

$$
2W_1^{\text{point}} = \frac{Q^2}{2m^2} \delta\left(v - \frac{Q^2}{2m}\right), \qquad 2mW_2^{\text{point}} = \delta\left(v - \frac{Q^2}{2m}\right).
$$

below the dashed line in Fig. 8.5, where a (virtual) photon interacts with a proton.

$$
2mW_1^{\text{point}}(\nu, Q^2) = \frac{Q^2}{2m\nu} \delta\left(1 - \frac{Q^2}{2m\nu}\right),
$$

$$
\nu W_2^{\text{point}}(\nu, Q^2) = \delta\left(1 - \frac{Q^2}{2m\nu}\right).
$$

M.C. Kumar welastic = _ *Q2) ¹ 4M2 2M '*

within the proton provided A(:::::: *l/V- q2)* « IF.

$$
\bigg(\frac{G_E^2+\tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2}+2\tau G_M^2\sin^2\frac{\theta}{2}\bigg)\delta\bigg(\nu+\frac{q^2}{2M}\bigg),
$$

$$
Q^2 \equiv -q^2.
$$

w;point = *(p* -

$$
W_2(\nu, q^2)\cos^2\frac{\theta}{2}+2W_1(\nu, q^2)\sin^2\frac{\theta}{2}.
$$

$$
W_1^{\text{elastic}} = \frac{Q^2}{4M^2} G^2 (Q^2) \delta \left(\nu - \frac{Q^2}{2M} \right),
$$

$$
W_2^{\text{elastic}} = G^2 (Q^2) \delta \left(\nu - \frac{Q^2}{2M} \right).
$$

The role of the electron beam is simply that it is responsible for the presence of

Structure functions continuation introduced the proton mass instead of the quark mass to define the dimensionless **variable** *w***. The presence of free presence of free** α **free** α **fact that the inelastic that the inelast** structure functions are independent of *Q2* at a given value of *w* [see (9.5)]. This is

that is, at large *Q2, inelastic* electron-proton scattering is viewed simply as *elastic*

equivalent to the onset of sin-4(*012)* behavior for large momentum transfers in

$$
2mW_{1}^{\text{point}}(\nu, Q^{2}) = \frac{Q^{2}}{2m\nu} \delta\left(1 - \frac{Q^{2}}{2m\nu}\right),
$$
\n
$$
\nu W_{2}^{\text{point}}(\nu, Q^{2}) = \delta\left(1 - \frac{Q^{2}}{2m\nu}\right).
$$
\n
$$
MW_{1}(\nu, Q^{2}) \rightarrow F_{1}(\omega),
$$
\n
$$
MW_{1}(\nu, Q^{2}) \rightarrow F_{2}(\omega),
$$
\n
$$
\nu W_{2}(\nu, Q^{2}) \rightarrow F_{2}(\omega),
$$
\n
$$
\omega = \frac{2q \cdot p}{Q^{2}} = \frac{2M\nu}{Q^{2}}.
$$

Structure functions contd. latter do not interact with the photon, of course). They can each carry a different fraction *x* of the parent proton's momentum and energy. We introduce the parton momentum distribution

9.2 Partons and Bjorken Scaling

$$
f_i(x) = \frac{1}{p} \left(\frac{1}{\sqrt{1-x^2}} \right)^{x} (1-x)^p
$$

$$
\sum_{i'} \int dx \, xf_{i'}(x) = 1.
$$

Here, *if* sums over all the partons, not just the charged ones i which interact with

$$
\sum_{i'}\int dx\,xf_{i'}(x)=1.
$$

Both the proton and its parton progeny move along the *z* axis (i.e., *PT* = 0) with

Proton	Parton
\n \downarrow \n	\n $W_2(\nu, Q^2) \rightarrow F_2(x) = \sum_i e_i^2 x f_i(x),$ \n
\n $MW_1(\nu, Q^2) \rightarrow F_1(x) = \frac{1}{2x} F_2(x),$ \n	

$$
x=\frac{1}{\omega}=\frac{Q^2}{2M\nu}.
$$

PT= 0 *PT=* 0

Proton	Parton				
\n \downarrow \n	\n \downarrow \n	\n \downarrow \n			
\n \downarrow \n	\n \downarrow \n	\n $MW_1(\nu, Q^2) \rightarrow F_1(x) = \frac{1}{2x} F_2(x),$ \n			
\n \downarrow \n	\n \downarrow \n	\n $MW_1(\nu, Q^2) \rightarrow F_1(x) = \frac{1}{2x} F_2(x),$ \n			
\n Momentum\n p_L \n	\n $x p_L$ \n	\n $x p_L$ \n	\n \downarrow \n	\n \downarrow \n	\n $MW_1(\nu, Q^2) \rightarrow F_1(x) = \frac{1}{2x} F_2(x),$ \n
\n Mass\n M \n	\n $m = (x^2 E^2 - x^2 p_L^2)^{1/2} = xM$ \n	\n $x = \frac{1}{\sqrt{2}} = \frac{Q^2}{2M}.$ \n			

Proton Parton

which describes the probability that the struck parton i carries a fraction *x* of the

Proton Parton

Colour and all that

 $e^-e^+ \rightarrow \mu^-\mu^+$ $\Rightarrow (e^-e^+) \Rightarrow (\overline{q}\overline{q}) \rightarrow 250ts$ $(\sqrt[n]{\tilde{\psi}}) \rightarrow \hat{\psi}\sqrt[n]{\tilde{\psi}}$, $\hat{\psi}_2\hat{\psi}_2... \hat{\psi}_n\hat{\psi}_n$ $e^-e^- \rightarrow \hat{\psi}\sqrt[n]{\tilde{\psi}} \rightarrow \tilde{\psi}_0$ $e^-e^+ \rightarrow \mu^- \mu^+ \rightarrow \sigma_{\nu}$ $rac{Q_{(1)}}{Q_{(2)}} \sim N \frac{\sum_{q} e_{q}^{2}}{Q_{q}^{2}}$ $\begin{array}{c}\n & \mathbf{Q}=-1 \\
 & \mathbf{W} \rightarrow 3 \\
 & \mathbf{Q}_{\mathbf{V}} = \mathbf{Q}_{\mathbf{V}} \\
 & \mathbf{W} = \mathbf{W}_{3}, \mathbf{C}, \mathbf{t}\n\end{array}$ $\left(\gamma ,P:u,(P) \right)$ $(i=\beta,\beta,\beta)$ $d=-1/3,3,5$ $Z(x_i f(x_i) dx_i = 1 | u,\overline{u}, d,\overline{d}, 55, c,\overline{c}, b,\overline{b}$ \neg proton μ μ α $\int [u(x_i) - \overline{u}(x_i)]dx_i = 2$ $\int\int d(x_i) - \overline{d}(x_i) dx_i = 1$ $\int [6(x_i) - 5(x_i)] dx_i = 0$

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Gluons …

Gluons - Sni[u(x) ⁺ $\widehat{u}(x_i) + d(x_i) + d(x_i)$.
مبر $\frac{1}{1 - -1} \frac{1}{\alpha x} = 1.$ $\int x_i \left[u(x_i) + \overline{u}(x) \right] dx_i \sim 0.$ $\frac{1}{1+u(x)}\frac{1}{x}dx$ ~ 0.38 $\int x_i \left[d(x_i) + d(x_i) \right] dx_i \sim 0.16$ 0.54 $\int x_i \left[d(x_i) + d(x_i) \right] dx_i \sim 0.16$ $\frac{1}{2} \frac{\int d(x_i) + d(x_i)}{x_i}$ $SO(1) + W(0)$ and $SO(1)$
 $SO(1)$ \longrightarrow $Eq \rightarrow G[$ N e a jets .
2 jets .
2 jets . jet = ^v + \sim $\sqrt{\frac{1}{\sqrt{1-\frac{1}{2}}}$ $Gluons...$
 $\frac{C_{\gamma}\left\{u\right\}}{x_{i}\left[u\left(x_{i}\right)+1\right]}$ $u(x) = -x$ (12) $-\frac{1}{4\pi(x)}\frac{1}{(x^{2}+1)^{2}}$ $\sqrt{2}$ 54 \overline{h} 50 $\sqrt{ }$ Eg Suang jet = ^v -nf ⁺ n suset³ [↑] $e^-e^+ \rightarrow 3$ jets \Rightarrow jets jet · et-wzg e \mathbb{Z} e^{+} jet \mathbf{a} 30 . Sci [dInD ⁺ (i)] dai~0. $\sqrt{2}$ $e^-e^+ \rightarrow 3$ jets. \Rightarrow j N $\frac{d(x_i)}{1 - 1}$ $+$ $=$ jets . $\frac{2}{\sqrt{2}}$ $\int dx_i \sim 0.38$ 0.54 $e^{\frac{1}{2}}$ suret the set ifonlyquartedated $\begin{picture}(180,190)(0,0) \put(0,0){\line(1,0){15}} \put(10,0){\line(1,0){15}} \put(10,$ jet $e^-e^+ \rightarrow \sqrt{\gamma} + \gamma$

e

shares colored:

jet poznata za poznata
Za poznata za poznata

et-wzg

·

suset³ [↑] e-et-3 jets - - jets -bunch ofhad ifonlyquartedated quark
ze Produ 24 if only quark
sette faire are Product jet N e⁻e + $\frac{1}{\sqrt[4]{y}} \rightarrow$ 2 jets. $e^-e^+ \rightarrow \gamma^*$ $\frac{v}{v}$ 2 iets 3 4 setz ? if only quart Jety if only quark
Jety faire are Produce s in the set of $\frac{1}{2}$

jet

jet

e et al. et
Et al. et al

 $\sin\rightarrow$ colored:

q -> ² jets .

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Gluons ...

QCD Lagrangian (simpler form)

ngian (simpler:
QCD hagrangion
In a Gimplex ACD Lagrangion In a simpler form, it Can be written in a similar way as in GED, $\int_{\omega} = \frac{1}{\mu} F_{\mu\nu}^{\alpha} F^{\mu\nu,\alpha}$ $+\overline{\psi}\left[i\gamma^{\mu}\mathcal{D}_{\mu}-m\right]\psi$ $D_{\mu} = \partial_{\mu} - i g A_{\mu}^{a} t^{a}$ $F_{\mu\nu}^{\alpha}$ = $\partial_{\mu}A_{\nu}^{\alpha}$ - $\partial_{\nu}A_{\mu}^{\alpha}$ + $9f^{abc}A_{\nu}^bA_{\nu}^c$ gabe = structure constants (feelly anti-symmetric) & Cfally anti-symmetric)
g -> Strong coupling constant

Feynman rules

