# QCD Lecture-2

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# Outline

Lecture -2

**Naive parton model.**

→ $e^- \mu^-$ Scattering

→ $e^- P$ elastic scattering

↳ Form Factors

→ $e^- P$ Inelastic scattering

↳ structure functions

↳ Björken Scaling

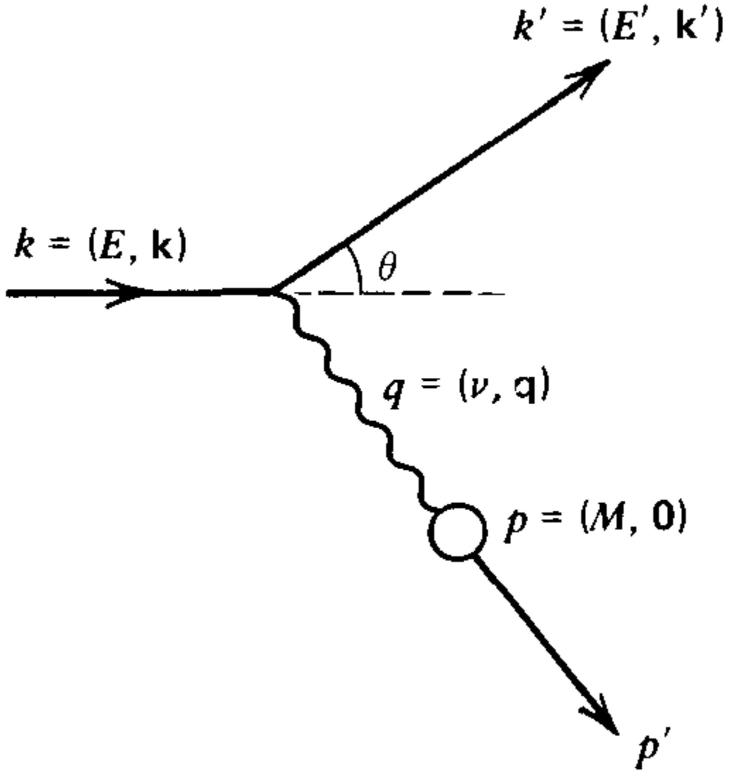
↳ parton Distribution functions

↳ Gluons

**QCD Improved Parton model**

Beyond the scope of this lecture.

# Electron muon scattering




$$\nu \equiv E - E' = -\frac{q^2}{2M}$$

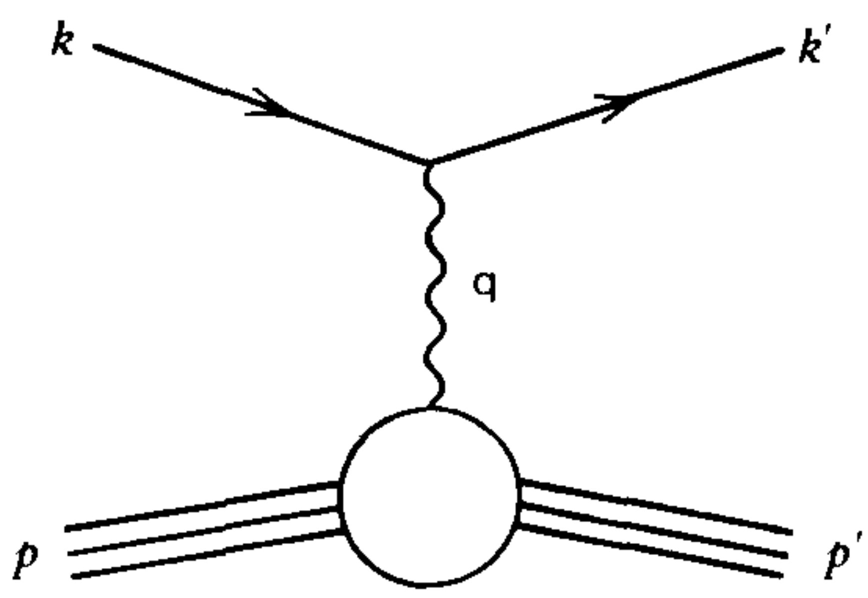
$$\frac{d\sigma}{dE'\,d\Omega} = \frac{(2\alpha E')^2}{q^4}\left\{\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right\}\delta\left(\nu + \frac{q^2}{2M}\right)$$

$$\frac{E'}{E} = \frac{1}{1 + \frac{2E}{M}\sin^2\frac{\theta}{2}}$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab.}} = \left(\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\right)\frac{E'}{E}\left\{\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right\}.$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab.}} = \left(\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\right)\frac{E'}{E}\cos^2\frac{\theta}{2}$$

# Electron proton elastic scattering



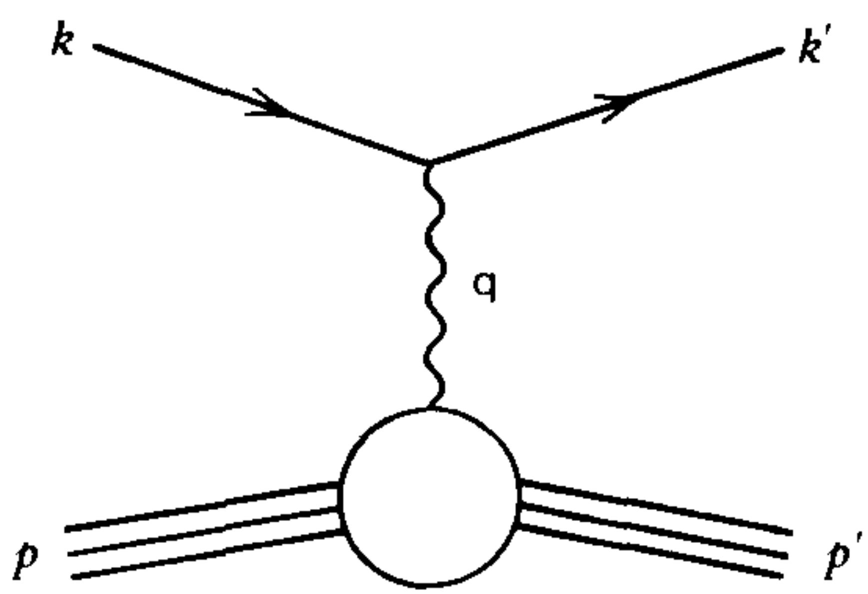

$$(1+\kappa)e/2M,$$

$$\left[F_1(q^2)\gamma^\mu + \frac{\kappa}{2M}F_2(q^2)\, i\sigma^{\mu\nu}q_\nu\right]$$

$$d\sigma \sim L^e_{\mu\nu}(L^p)^{\mu\nu}$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{\rm lab} = \left(\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\right)\frac{E'}{E}\left\{\left(F_1^2 - \frac{\kappa^2 q^2}{4M^2}F_2^2\right)\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}(F_1+\kappa F_2)^2\sin^2\frac{\theta}{2}\right\}$$

# Electron proton elastic scattering



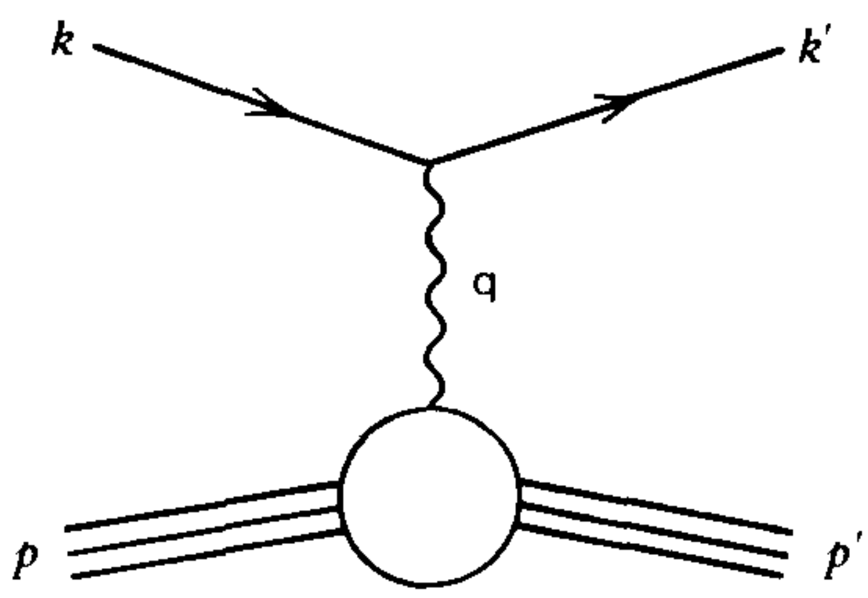

$$G_E \equiv F_1 + \frac{\kappa q^2}{4M^2} F_2$$

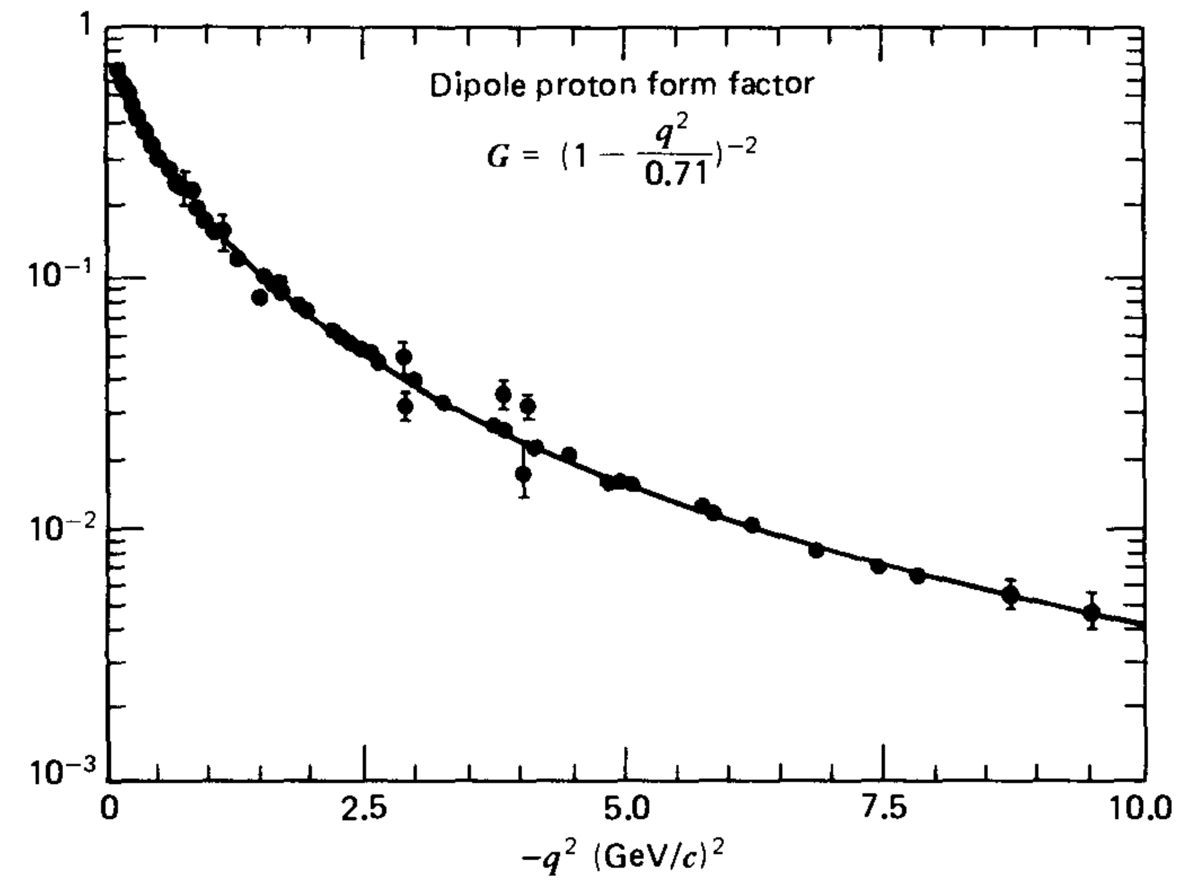
$$G_M \equiv F_1 + \kappa F_2,$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab}} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \frac{E'}{E} \left( \frac{G_E^2 + \tau G_M^2}{1+\tau} \cos^2 \frac{\theta}{2} + 2\tau G_M^2 \sin^2 \frac{\theta}{2} \right)$$

$$\tau \equiv -q^2/4M^2.$$

# Electron proton elastic scattering



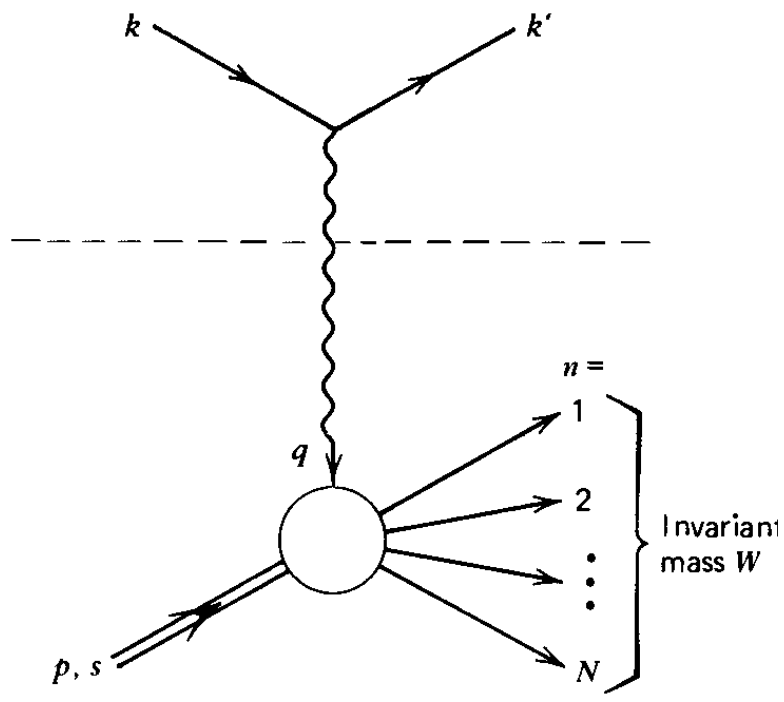

$$G_E(q^2) \approx \left(1 - \frac{q^2}{0.71}\right)^{-2}$$

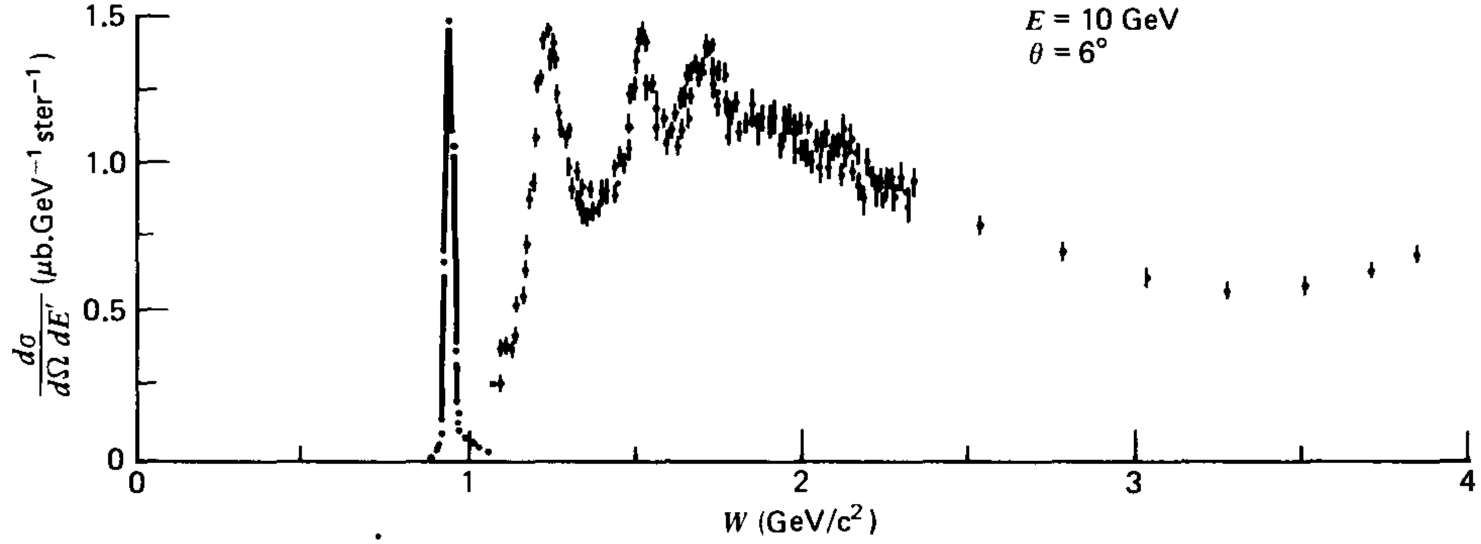
$$\langle r^2 \rangle = 6\left(\frac{dG_E(q^2)}{dq^2}\right)_{q^2=0} = (0.81 \times 10^{-13}\ \text{cm})^2$$

mean square proton charge radius

**Ref : Halzen and Martin**

# Electron proton inelastic scattering




$$d\sigma \sim L^{e}_{\mu\nu}(L^{p})^{\mu\nu}$$

$$d\sigma \sim L^{e}_{\mu\nu}W^{\mu\nu}$$

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{M^2}p^{\mu}p^{\nu} + \frac{W_4}{M^2}q^{\mu}q^{\nu} + \frac{W_5}{M^2}(p^{\mu}q^{\nu} + q^{\mu}p^{\nu})$$

# Electron proton inelastic scattering

$$q^\mu L^e_{\mu\nu} = q^\nu L^e_{\mu\nu} = 0.$$

$$W_5 = -\frac{p\cdot q}{q^2}W_2,$$

$$W_4 = \left(\frac{p\cdot q}{q^2}\right)^2 W_2 + \frac{M^2}{q^2}W_1$$

$$q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0.$$

$$W^{\mu\nu} = W_1\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + W_2\frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right)$$

$q^2$ and $\nu \equiv \dfrac{p\cdot q}{M}.$

$$x = \frac{-q^2}{2p\cdot q} = \frac{-q^2}{2M\nu}, \qquad y = \frac{p\cdot q}{p\cdot k}$$

$$\left.\frac{d\sigma}{dE'\,d\Omega}\right|_{\text{lab}} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\left\{W_2(\nu,q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu,q^2)\sin^2\frac{\theta}{2}\right\},$$

# Electron proton inelastic scattering

$$\left(\cos^2\frac{\theta}{2} - \frac{q^2}{2m^2}\sin^2\frac{\theta}{2}\right)\delta\left(\nu + \frac{q^2}{2m}\right).$$

Electron muon scattering

$$\left(\frac{G_E^2 + \tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2} + 2\tau G_M^2\sin^2\frac{\theta}{2}\right)\delta\left(\nu + \frac{q^2}{2M}\right),$$

Electron proton elastic scattering

$$W_2(\nu, q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu, q^2)\sin^2\frac{\theta}{2}.$$

Electron proton inelastic scattering

$$2W_1^{\text{point}} = \frac{Q^2}{2m^2}\delta\left(\nu - \frac{Q^2}{2m}\right),$$

$$W_2^{\text{point}} = \delta\left(\nu - \frac{Q^2}{2m}\right).$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\frac{E'}{E}\left(\quad\right)$$

# For point like particles

$$\left(\cos^2\frac{\theta}{2} - \frac{q^2}{2m^2}\sin^2\frac{\theta}{2}\right)\delta\left(\nu + \frac{q^2}{2m}\right).$$

$$Q^2 \equiv -q^2.$$

$$W_2(\nu, q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu, q^2)\sin^2\frac{\theta}{2}.$$

$$2W_1^{\text{point}} = \frac{Q^2}{2m^2}\delta\left(\nu - \frac{Q^2}{2m}\right),$$

$$W_2^{\text{point}} = \delta\left(\nu - \frac{Q^2}{2m}\right).$$

$$2mW_1^{\text{point}}(\nu, Q^2) = \frac{Q^2}{2m\nu}\delta\left(1 - \frac{Q^2}{2m\nu}\right),$$

$$\nu W_2^{\text{point}}(\nu, Q^2) = \delta\left(1 - \frac{Q^2}{2m\nu}\right).$$

# For extended objects

$$\left( \frac{G_E^2 + \tau G_M^2}{1+\tau} \cos^2 \frac{\theta}{2} + 2\tau G_M^2 \sin^2 \frac{\theta}{2} \right) \delta\left( \nu + \frac{q^2}{2M} \right),$$

$$Q^2 \equiv -q^2.$$

$$W_2(\nu, q^2) \cos^2 \frac{\theta}{2} + 2W_1(\nu, q^2) \sin^2 \frac{\theta}{2}.$$

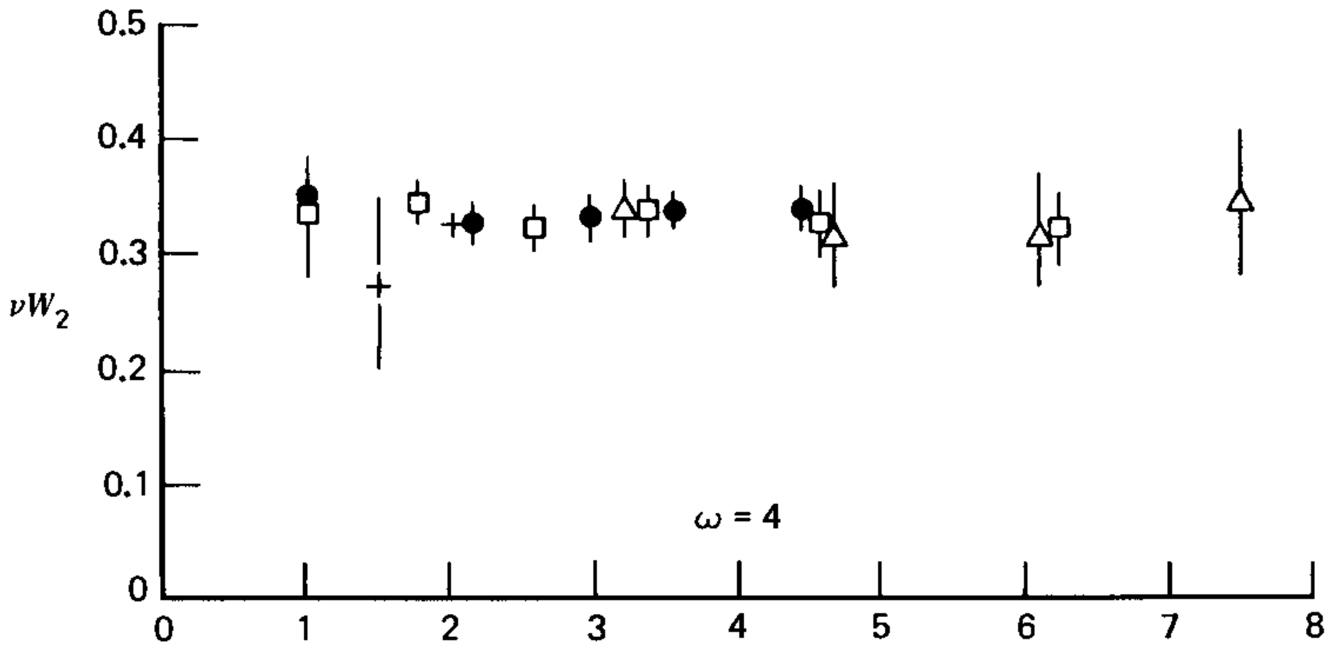
$$W_1^{\text{elastic}} = \frac{Q^2}{4M^2} G^2(Q^2) \delta\left( \nu - \frac{Q^2}{2M} \right),$$

$$W_2^{\text{elastic}} = G^2(Q^2) \delta\left( \nu - \frac{Q^2}{2M} \right).$$

# Structure functions continuation

$$2mW_1^{\text{point}}(\nu, Q^2) = \frac{Q^2}{2m\nu}\delta\left(1 - \frac{Q^2}{2m\nu}\right),$$

$$\nu W_2^{\text{point}}(\nu, Q^2) = \delta\left(1 - \frac{Q^2}{2m\nu}\right).$$

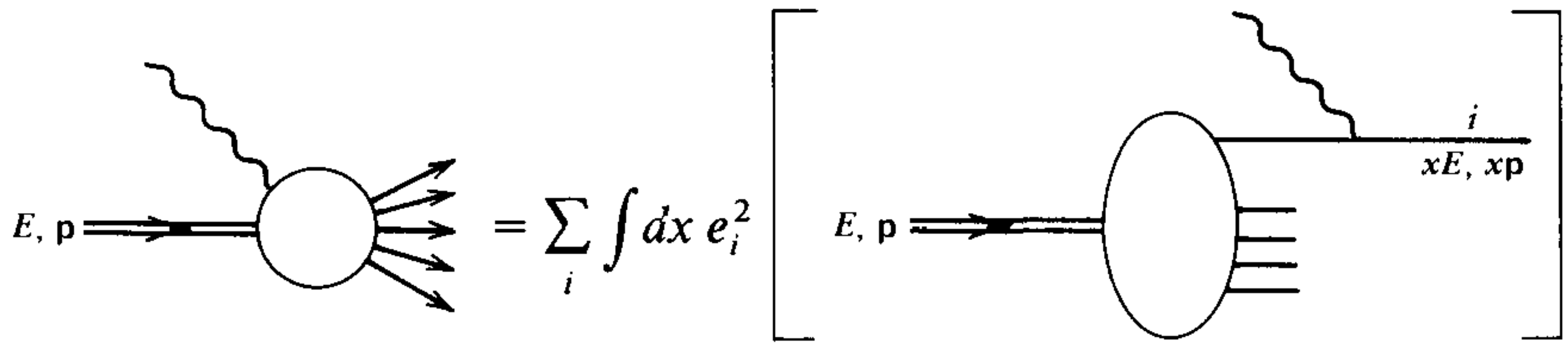


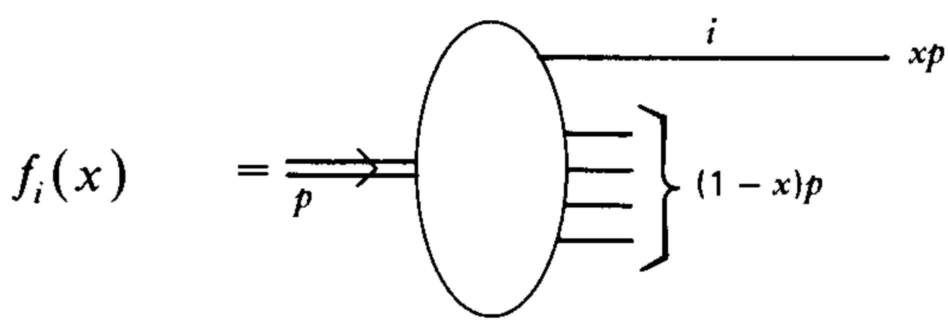

$$MW_1(\nu, Q^2) \xrightarrow[\text{large } Q^2]{} F_1(\omega),$$

$$\nu W_2(\nu, Q^2) \xrightarrow[\text{large } Q^2]{} F_2(\omega),$$

$$\omega = \frac{2q \cdot p}{Q^2} = \frac{2M\nu}{Q^2}.$$

# Structure functions contd.




$$\sum_{i'} \int dx\, x f_{i'}(x) = 1.$$

| | Proton $\downarrow$ | Parton $\downarrow$ |
|---|---|---|
| Energy | $E$ | $xE$ |
| Momentum | $p_L$ | $xp_L$ |
| | $p_T = 0$ | $p_T = 0$ |
| Mass | $M$ | $m = (x^2E^2 - x^2p_L^2)^{1/2} = xM$ |

$$\nu W_2(\nu, Q^2) \to F_2(x) = \sum_i e_i^2 x\, f_i(x),$$

$$M W_1(\nu, Q^2) \to F_1(x) = \frac{1}{2x} F_2(x),$$

$$x = \frac{1}{\omega} = \frac{Q^2}{2M\nu}.$$

# Colour and all that

$$e^-e^+ \to \mu^-\mu^+$$
$$\to e^-e^+$$
$$\Rightarrow (e^-e^+) \to (q\bar{q}) \to 2 \text{ jets.}$$

$$(q\bar{q}) \to Q_1\bar{Q}_1, Q_2\bar{Q}_2 \ldots Q_n\bar{Q}_n \qquad e^-e^- \to q_i\bar{q}_i \to \sigma_{(1)}$$
$$e^-e^+ \to \mu^-\mu^+ \to \sigma_{(2)}$$

$$\boxed{\frac{\sigma_{(1)}}{\sigma_{(2)}} \sim N \sum_q e_q^2}$$

$$\boxed{N \to 3}$$

$Q_\mu = -1$

$Q_q = e_q$.

$e^-, p$ : $u(P)$

$q, p$ : $u_i(P)$ $(i = R, G, B)$.

$u = +2/3$, $c$, $t$

$d = -1/3$, $s$, $b$

$$\boxed{\sum_i x_i f(x_i)\, dx_i = 1}$$

$u, \bar{u}, d, \bar{d}, s\bar{s}, c, \bar{c}, b\, \bar{b}$

proton: $(u\ u\ d)$

$$\int [u(x_i) - \bar{u}(x_i)]\, dx_i = 2$$

$$\int [d(x_i) - \bar{d}(x_i)]\, dx_i = 1 \qquad \int [s(x_i) - \bar{s}(x_i)]\, dx_i = 0$$

# Gluons …

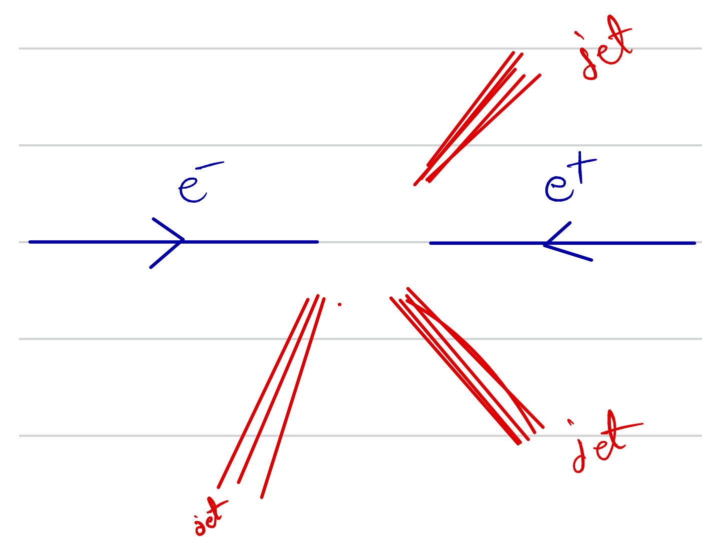
## Gluons

$$\int x_i\left[u(x_i)+\bar{u}(x_i)+d(x_i)+\bar{d}(x_i) + \cdots\right] dx_i = 1$$

$$\left.\begin{aligned}\int x_i\left[u(x_i)+\bar{u}(x_i)\right] dx_i &\sim 0.38\\ \int x_i\left[d(x_i)+\bar{d}(x_i)\right] dx_i &\sim 0.16\end{aligned}\right\} 0.54$$

50% → Eg → Gluons

$e^-e^+ \to 3$ jets. ⇒ jets




$e^-e^+ \to q\bar{q}$ ✓ → 2 jets.

$e^-e^+ \to \gamma^* \to nf + n\bar{f}$

→ 4 jets
→ 2n jets. } if only quark pairs are produced.

$e^-e^+ \to q\bar{q} + g$

gluon → colored.

# Gluons …

$$g = R\bar{G},\ R\bar{R} + B\bar{B} + G\bar{G}$$

$$+ R\bar{B},\ B\bar{R} \text{ --- ---}$$

$N = 3.$

R G B

$$\psi_i(x) \rightarrow e^{i\tau^a_{ii}\theta(x)}\, \psi_i(x)$$

$$\tau^a \rightarrow SU(3).$$

$$SU(N) \rightarrow \underline{N^2 - 1}$$

$$\Downarrow$$

8 – gluons.

Generators $\Rightarrow$ Gell-mann matrices.

$$a = 1, 2, \ldots, 8.$$

# QCD Lagrangian (simpler form)

QCD Lagrangian

In a simpler form,

it can be written in a similar way as in QED,

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu,a} + \bar{\psi}\left[i\gamma^\mu D_\mu - m\right]\psi$$

$$D_\mu = \partial_\mu - i g A^a_\mu t^a$$

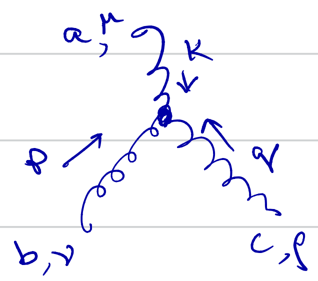
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$f^{abc}$ = structure constants (fully anti-symmetric)

$g \rightarrow$ strong coupling constant.

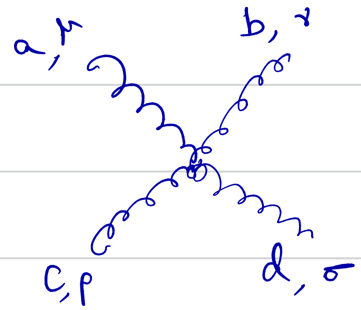
# Feynman rules

## QCD Feynman rules



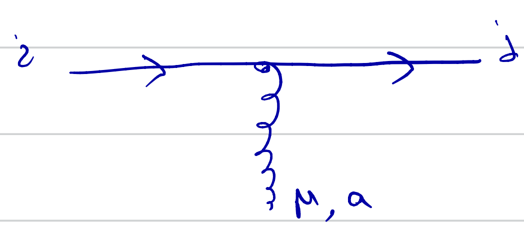

$$= g f^{abc}\left[g^{\mu\nu}(k-p)^{\rho} + g^{\nu\rho}(p-q)^{\mu} + g^{\rho\mu}(q-k)^{\nu}\right]$$

(3 - gluon vertex)

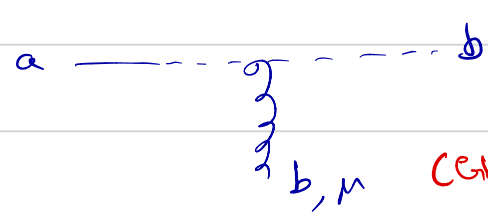



$$= -ig^2\left[f^{abe}f^{cde}\left(g^{\mu\rho}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\rho}\right) + f^{ace}f^{bde}\left(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\sigma}g^{\nu\rho}\right) + f^{ade}f^{bce}\left(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\rho}g^{\nu\sigma}\right)\right]$$

(4 - gluon vertex)




$$i g t^{a} \gamma^{\mu}$$

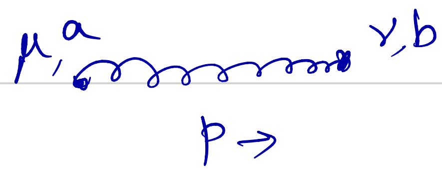



(Ghost - gluon vertex).

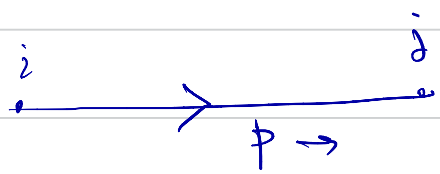
$$= -g f^{abc} p^{\mu}$$

# Feynman rules

$$= \frac{i\,\delta^{ab}}{p^2 + i\epsilon}$$




$$\frac{-i\,g^{\mu\nu}\,\delta^{ab}}{p^2 + i\epsilon}$$




$$\frac{i\,(\not p + m)\,\delta_{ij}}{p^2 - m^2 + i\epsilon}$$

$$\sum_s u_i^{(s)}(p)\,\bar{u}_j(p) = \delta_{ij}(\not p + m)$$

$$\boxed{\not p = p^\mu \gamma_\mu}$$

# Thank You