Perturbative QCD

Lecture-1

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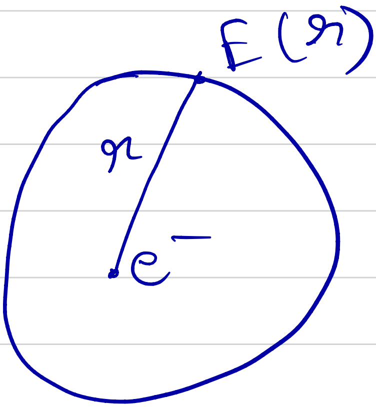
→ QED
- → Basic ideas of EM field
- → " " Dirac field
- → Gauge invariance
- → QED Lagrangian
- → Feynman rules
- → Example Processes.

Introduction

→ Sources & Forces.

→ classical Electromagnetism.



$$E(r) = \frac{e^2}{4\pi\epsilon_0 r^2}$$

$$\boxed{Q = -1}$$

$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi} \frac{I\,dl\,\sin\theta}{r^2}$$

Maxwell's equations

$\vec{E}, \vec{B}$ are forces

$\vec{E} = (E_x, E_y, E_z)$

e^-, I ⇒ sources

$\vec{B} = (B_x, B_y, B_z)$.

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \Rightarrow \vec{B} = \vec{\nabla}\times\vec{A} \longrightarrow (1)$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \Rightarrow \vec{\nabla}\times\left(\vec{E}+\frac{\partial\vec{A}}{\partial t}\right) = 0$$

$$\Rightarrow \vec{E}+\frac{\partial\vec{A}}{\partial t} = -\vec{\nabla}\phi$$

$$\Rightarrow \vec{E} = -\vec{\nabla}\phi - \frac{\partial\vec{A}}{\partial t} \longrightarrow (2)$$

✓

$$\left.\begin{array}{l}\vec{A} \longrightarrow \vec{A}+\vec{\nabla} f \\ \phi \longrightarrow \phi - \dfrac{\partial f}{\partial t}\end{array}\right\} \Rightarrow \text{Same } \vec{E} \;\&\; \vec{B}$$

Gauge transformations.

Both ϕ & $\vec{A}$ are not Physical whereas $\vec{E}$ and $\vec{B}$ are Physical.

$$(\vec{E},\vec{B}) \rightarrow (\phi,\vec{A}) = (A^0,\vec{A}) = A^\mu \quad (\text{E. M. field}).$$

$$\mathcal{L}_{em} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\boxed{A_\mu \rightarrow A_\mu + \partial_\mu f}$$

$$\Downarrow$$

$F_{\mu\nu} \rightarrow$ invariant.

Quantum level $\Rightarrow$ Fields $\Rightarrow$ operators!

$A_\mu(x) \longrightarrow \epsilon_\mu(k)\, e^{ik\cdot x}$ → polarization vector.

All the matter fields are described by fermions. (spin-1/2)

→ Force carriers are by gauge bosons.

$$\mathcal{L}^{\text{free}}_{\text{Dirac}} = \bar{\psi}\left[i\gamma^\mu \partial_\mu - m\right]\psi \longrightarrow (3)$$

$$= \underbrace{\bar{\psi} i \gamma^\mu \partial_\mu \psi}_{\text{Kinetic}} - m\bar{\psi}\psi \rightarrow \text{mass}$$

ψ → Dirac field → 4-component vector.

$$\psi = \begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{pmatrix} \longrightarrow \underbrace{\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}}_{E>0}, \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix} \quad E<0$$

$u(p)$ $\quad$ $v(p)$

$\{\gamma_\mu, \gamma_\nu\}$, $\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 0$

Two fold degeneracy → $\boxed{\text{spin}}$

Does $\mathcal{L}_{\text{Dirac}}$ has any invariance under gauge transformations?

$$\psi(x) \to \psi'(x) = e^{-i\theta}\,\psi(x) \longrightarrow (4)$$

θ = constant $\Rightarrow$ Global gauge transformation

$\gamma_\mu \to$ Dirac matrices $\quad \gamma^\mu = (\gamma^0, \vec{\gamma})$, 4×4 matrices

$\bar{\psi}(x) = \psi^\dagger(x)\,\gamma^0$

Under (4), $\mathcal{L}_{Dirac}$ is invariant. (Trivial)

What about the case when $\theta = \theta(x)$

i.e. more generally

$$\psi(x) = e^{-iQ\theta(x)}\,\psi(x)$$

Q = Generator, here it is a number.

$\theta(x)$ = local gauge parameter

i.e. it depends on the position "x".

One can see that $\mathcal{L}_D$ is not invariant under local gauge transformations

Intutive way.

Equilibrium

will they remain like this?

$$\mathcal{L}_D = \bar{\psi}\left[i\gamma^\mu \partial_\mu - m\right]\psi$$

$$\Big\downarrow \; \psi(x) \to \psi'(x) = e^{-iQ\theta(x)}\psi(x).$$

$$\partial_\mu \psi'(x) = e^{-iQ\theta(x)}\partial_\mu \psi(x) + e^{-iQ\theta(x)}\psi(x)\left[-iQ\partial_\mu \theta\right]$$

$$\therefore \mathcal{L}'_D = \bar{\psi}'\left[i\gamma^\mu \partial_\mu\right]\psi' - m\bar{\psi}'\psi'$$

$$= \bar{\psi}\left[i\gamma^\mu \partial_\mu\right]\psi + \boxed{Q\,\partial_\mu \theta\left[\bar{\psi}\gamma^\mu \psi\right]} - m\bar{\psi}\psi$$

Lets consider an interaction term

$$\mathcal{L}_{int} = -Q\bar{\psi}\gamma^\mu A_\mu \psi \qquad \text{and } A_\mu \longrightarrow A_\mu + \partial_\mu \theta(x)$$

$$\mathcal{L}'_{int} = -Q\bar{\psi}\gamma^\mu A_\mu \psi \boxed{-Q\bar{\psi}\gamma^\mu \psi\left[\partial_\mu \theta\right]}$$

$$\therefore \mathcal{L}_D + \mathcal{L}_{gauge} + \mathcal{L}_{int} = \mathcal{L}_{QED}$$

is invariant under gauge transformation!

$$\psi(x) \longrightarrow e^{-iQ\theta(x)}\psi(x) \text{ and } A_\mu \longrightarrow A_\mu + \partial_\mu \theta(x).$$

In Summary,

$$\mathcal{L}_{QED} = \bar{\psi}\left[i\gamma^{\mu}D_{\mu} - m\right]\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$D_{\mu} = \partial_{\mu} - ieA_{\mu}$: Co-variant derivative.

→ The quantization of both the Dirac and Gauge fields give the respective field quanta ⇒ Fermions and Photons.

→ For Quantization, refer to and Standard Field Theory text book.

→ Interactions and 2-Point correlations can be described by Feynman rules.

Feynman Rules for QED

Incoming Fermion: (towards interaction point) $\quad p \rightarrow \quad u(p)$

outgoing Fermion $\quad \bar{u}(p)$

Incoming anti-fermion $\quad p \rightarrow \quad \bar{v}(p)$

outgoing anti-fermion $\quad p \rightarrow \quad v(p)$

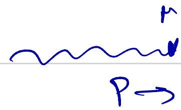
propagator : $\quad p \rightarrow \quad \dfrac{i(\not{p}+m)}{(p^2-m^2+i\epsilon)}$

$u(p)$: For +ve energy $(E>0)$ solution
$v(p)$: " -ve " $(E<0)$ solution.

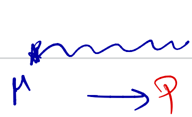
Polarization Sum: $\sum_s u_s(p)\,\bar{u}_s(p) = (\not{p}+m)$

$$\boxed{\not{p} = p_\mu \gamma^\mu}$$

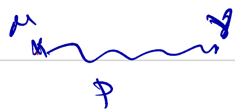
Photons:

Incoming photon: 

$\epsilon^\mu(p)$

outgoing photon : 

$\epsilon^{*\mu}(p)$

propagator : 

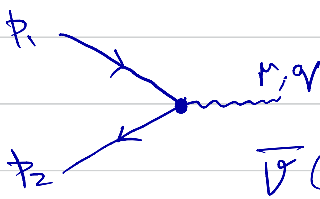
: $\dfrac{-ig^{\mu\nu}}{p^2+i\epsilon}$

Sum over polarizations

Feynman gauge

$$\boxed{\sum_s \epsilon_s^\mu(p)\,\epsilon_s^{*\nu}(p) = -g^{\mu\nu}}$$

Interaction:

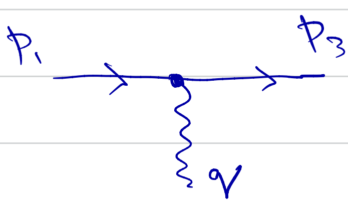


: $\boxed{-iQ\gamma^\mu}$

$\bar{v}(p_2)[ie\gamma^\mu]u(p_1)$ $\epsilon_\mu(q)$

$Q=-e$ for electron:

$\Rightarrow \boxed{ie\gamma^\mu}$



$\bar{u}(p_3)[ie\gamma^\mu]u(p_1)\cdot\epsilon_\mu(q)$

Strong Interactions

→ Historically it goes back to the structure of atom

⇒ Rutherford scattering of α- particles on the thin gold foil

⇒ most of the atom is empty
(α- particles go through the gold foil)

⇒ centre of the atom has a dense core (positively charged protons)
(some α- particles got deflected by more than 90°).

protons are held together inspite of large coulomb repulsions due to the strong attractive forces.

⇒ Nuclear strong force among nucleons
(This is not QCD).

Note: (1) Scattering experiments are very helpful in probing the structure

(2) Strong forces are short range forces.

(3) What about nuclears (protons, neutrons)
$\Rightarrow$ Are they elementary.

(4) Do they have spin?

(5) Answer is again from the scattering experiments.

Exercise:

Scattering of two particles in
(i) the c.o.m. frame
(ii) the lab frame.

Ex: $e^- \mu^-$ Scattering:

$$e^-(P_1) + \mu^-(P_2) \longrightarrow e^-(P_3) + \mu^-(P_4).$$

muon at rest $\Rightarrow$ $P_2 = (E, \vec{P}) = (M, \vec{0})$.

Imp. Points to note:

→ QFT is a mixture of special Theory of relativity and Quantum mechanics.

→ S.T.R. → lot of 'c' terms
→ Q.M. → lot of '$\hbar$' terms.

Natural units:

$\hbar = 1 \quad c = 1$

e.g. ① $E^2 = \vec{P}^2 c^2 + m^2 c^4 \Rightarrow E^2 = \vec{P}^2 + m_0^2$

② $E = h\nu = \hbar\omega = \omega$ ④ $E = m_0 c^2 \Rightarrow E = m_0$

③ $\beta = v/c$, $\gamma = \dfrac{1}{\sqrt{1-\beta^2}} = \dfrac{1}{\sqrt{1-v^2}}$

→ High energy limit: K.E. >> Rest mass energy

$\Rightarrow \vec{P}^2 \gg m_0^2 \Rightarrow E^2 \simeq \vec{P}^2 \Rightarrow E = |\vec{P}|$.

→ massless limit

→ Four vector:

$$p^\mu = (E/c, \vec{p}) = (E, \vec{p})$$

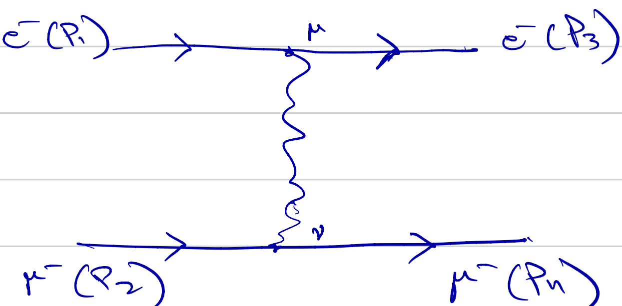
e.g. high energy particle moving along z-axis:

$$\vec{p} = (0, 0, |\vec{p}|) = (0, 0, E)$$

$$\therefore \; p^\mu = E(1, 0, 0, 1).$$

→ $e^- \mu^-$ Scattering process:

$$e^-(P_1) + \mu^-(P_2) \longrightarrow e^-(P_3) + \mu^-(P_4).$$



$$s = (P_1 + P_2)^2 = (P_3 + P_4)^2$$

$$t = (P_1 - P_3)^2 = (P_2 - P_4)^2$$

$$u = (P_1 - P_4)^2 = (P_2 - P_3)^2$$